Analytical Solution for Cold-air-drainage flow On Sloping Forest

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Cold-air-drainage Flow

Source: C.David Whiteman “Mountain Meterology”
Cold-air-drainage Flow

- **Mitigate the nocturnal UHI**
  - Kitada, 1998; Ohashi and Kida, 2002

- **Disperse the urban pollution**
  - Baumbach and Vogt, 1999; Egan, 1984; Lu and Turco, 1994

- **Influence the nocturnal ecosystem-atmosphere exchange**
  - Lee and Hu, 2002; Turnipseed et al, 2003; Yi et al, 2000
Hong Kong Island

Source: www.gearthblog.com
Physical model
Analytical Models

• Prandtl model
  – One dimensional, but gives the detailed structure of the flow profile

• Hydraulic model
  – Only provide the layer-averaged characteristic scales of flow parameters, i.e., velocity, momentum thickness, buoyancy deficit
Vegetation on Slope Flow

- Few studies address this problem
  - Bergen, 1969; Devito and Miller, 1983

- Katabatic flow can occur both within and above the tree canopies
  - Komatsu et al, 2003; Devito and Miller, 1983; Pypker, 2007

- Understand the flow structure can help to estimate the surface fluxes
Cold-air-drainage winds---Forest Canopy

katabatic wind above the canopy

katabatic wind in the canopy

Modified from Prof Tree’s PPT
Propose a simple analytical model by coupling both above and within tree canopies
Assumptions:

- One dimensional normal to the slope
- Constant deficit of potential temperature in the canopy
- Non-linear advection term is ignored in the momentum equation

$c_D$ --- drag coefficient;  
$a$ ---- leaf area density;  
LAI—leaf area index;  
$\Delta \theta$ --- deficit of the potential temperature;  
$h$ ---- canopy height
Flow Within Canopy
(non-uniform)

Momentum equation

\[
\frac{\partial u'w'}{\partial n} = g\beta\Delta \sin \alpha + c_DaN\overline{u}^2(n)
\]

Parameterizing the Reynolds stress (Yi, 2008):

\[
\tau(z)/\rho = -u'w'(z) = c_D(z)\overline{u}^2(z)
\]

\[
\frac{\partial(c_D(n)\overline{u}^2(n))}{\partial n} = g\beta\Delta \sin \alpha + c_D(n)a(n)\overline{u}^2(n)
\]

\[
U(n) = \left(\frac{c_D(0)}{c_D(n)}\right)U^2(0)e^{-[LAI-L(n)]} - \frac{g\beta\Delta \sin \alpha}{c_D(n)}\int_{n}^{0}e^{-[L(n')-L(n)]}dn'\right)^{1/2}
\]

\[
L(n) = \int_{-h}^{n}a(n')dn' \quad LAI = L(0)
\]
Flow Above Canopy

Prandtl Model

\[
\begin{aligned}
 g\beta \Delta \theta \sin \alpha &= k_m \frac{d^2 u(n)}{dn^2} \\
 \gamma u(n) \sin \alpha &= k_h \frac{d^2 \Delta \theta}{dn^2}
\end{aligned}
\]

\[u(n) = Ke^{-n/l} \left[ \Delta \theta_s \sin(n/l) - C' \cos(n/l) \right]\]

\[l = \left( \frac{4k_m k_h}{N^2 \sin^2 \alpha} \right)^{\frac{1}{4}}\]

\[K = \frac{g\beta}{N} \sqrt{\frac{k_h}{k_m}}\]
Coupling at Canopy Top

\[ \begin{align*}
\begin{cases}
\bar{u}(0)_{\text{in-canopy}} &= \bar{u}(0)_{\text{above-canopy}} \\
\Delta \theta\bigg|_{n=0,\text{in-canopy}} &= \Delta \theta\bigg|_{n=0,\text{above-canopy}} = \Delta \theta_s \\
d\bar{u}
\bigg|_{n=0,\text{in-canopy}} &= d\bar{u}
\bigg|_{n=0,\text{above-canopy}} \\
dn
\bigg|_{n=0,\text{in-canopy}} &= dn\bigg|_{n=0,\text{above-canopy}}
\end{cases}
\end{align*} \]

\[ \begin{align*}
\left\{ \begin{array}{l}
U(n) = -\left(\frac{C_D(0)}{C_D(n)}\right) K^2 C^2 e^{-[LAI - L(n)']} - \frac{g \beta \Delta \theta_s \sin \alpha}{c_D(n)} \int_{n}^{0} e^{-[L(n') - L(n)']} dn' \bigg)^{1/2} & \text{if } -h \leq n \leq 0 \\
U(n) = Ke^{-n/l} [\Delta \theta_s \sin(n/l) - C \cos(n/l)] & \text{if } n \geq 0
\end{array} \right.
\end{align*} \]
Validation and Discussion

Leaf area density
Velocity Profile

Minimum vertical exchange

Low-level jet

Super-stable layer

Minimum velocity and largest leaf area density

\[ Ri = \frac{g}{T} \left( \frac{\partial u}{\partial z} \right)^2 \rightarrow \infty \]
Sensitive Study
---Uniform Case

• Atmospheric Stability
• Slope angle
• Canopy morphology
Influence of Atmospheric Stability and Slope Angle

- \( \alpha = 5^\circ, \gamma = 2 \text{ K/km} \)
- \( \alpha = 5^\circ, \gamma = 4 \text{ K/km} \)
- \( \alpha = 10^\circ, \gamma = 2 \text{ K/km} \)

**Canopy top**

**Steeper slope**

**Weak stability**

**Strong stability**
Influence of Plant Canopy

![Graph showing the influence of plant canopy on velocity (m/s)]

- LAI = 2
- LAI = 4
- LAI = 10

Velocity (m/s)

Canopy Top

Legend:
- Blue: LAI = 2
- Red: LAI = 4
- Black: LAI = 10
Conclusions

• Analytical solution on cold-air-drainage winds by accounting for the influence of tree canopy is obtained.

• The effect of atmospheric stability and slope inclination is also investigated.

• The influence of different leaf area indexes is studied.
Thank you!